Fluid Mechanics

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# Basic Concepts

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| Heading | Definition |
| Viscous Fluid | A fluid is called a Viscous fluid if it has both Normal And shear Force, if non-viscous it doesn’t have a shear force value |
| Viscosity | Viscosity of a fluid is the property by which it exhibits resistance to the alteration of the form when upper plate moves in x dirn with certain velocity but the lower plate is stationary |
| Laminar Flow | A flow in which each particle traverses a definite curve, and the curve traced by any two particles never intersect. |
| Steady Flow | A flow in which we have that the properties such as temperature and pressure independent of time is called steady flow |
| Langrage Approach | We compare position at time with position at time i.e |
| Euler | We fix a point in space and look how fluid is behaving at that point |

# Important Theorems

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| Heading | Definition |
| Stokes Law | Let S be an open surface bounded by a closed curve C then |
| Gauss Divergence Theorem | Let S be a closed surface bounding a Volume V, and let be a unit vector perpendicular to the surface of V then |
| Green’s theorem | Let and be continuous differentiable function |

# Important Formulae’s

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| Heading | Definition’s | Formulae’s |
| Thermal Conductivity | Here is the conductive heat per unit area, k is the coefficient |  |
| Viscosity |  | Where u is constant of proportionality |
| Density, Velocity, Div |  |  |
| Material Derivative |  | Where is material derivative and is local derivative |
| Streamline Equation | Continuous Line of flow such that the tangent drawn at any point of streamline coincides with the dirn of motion of the fluid | Last equation is streamline at Point P |
| Path lines |  |  |
| Velocity and Acceleration of fluid particle |  |  |
| Equation of Continuity | Normal Form  LaGrange’s form | where J is Jacobian |
| Velocity potential (Scalar) |  |  |
| Rotational Flow Vector wz for 2-D flow |  | Where is velocity in y dirn and in x |
| Circulation | Scalar integral indicating measure of rotation along a closed curve  Flow across closed curve is called circulation around that curve | And this integral is >0 for anti-clockwise. W is the voriticity |
| Vorticity | Pseudo vector field describing spinning of fluid around a point |  |
| Vortex Line | A vortex Line is a curve drawn such that the tangent drawn at any point |  |
| Streak Lines | A streak line is defined to be locus of different particles passing through the same point | Let at any time be the fixed point  Hence, the streak line at time t’ would be |
| Linear Strain Rule |  | Where is dummy sum convention |
| Shear Stress or deformation of fluid element bcoz of Strains |  | denotesprinciple stress |
| Change in Density |  |  |
| Euler Equation of Motion |  |  |
| Lamb Hydrodynamics Equation |  |  |
| Generic Functions of stress and strain |  | Strain  Stress=  Shear= where is the angle |
| Properties of incompressible Liquids |  |  |
| Properties of Irrotational Fluids |  | Curl of Q |
| If force is conservative, we can use force field |  | Where is a force field |

# Stress and Strain

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| Heading | Definition’s | Formulae’s |
| Cauchy stress tensor | 2nd order tensor defining a stress at a point inside the material | Let Stress Vector be and be the unit vector in direction of T then |
| Moment of Force(torque) | It is the rotational equivalent of linear force and is equivalent to magnitude of Force and the perpendicular distance between force and axis | Where is the angle between r and F |
| Angular Momentum |  | *Where* |
| Moment of Inertia (analogy mass) | Defined to be as angular momentum by angular Velocity | Where L is the angular momentum |
| Stress Tensor and vector |  |  |
| Levi-Civia System |  | Where LHS has distinct and in ordered fashion |
| Cauchy First Law of Motion |  |  |
| Principal Stress |  | where  Where |
| Stress Derivative Tensor | is derivative stress tensor which tends to distort the body. tends to change its volume. |  |
| Bernoulli Equation | Here |  |
| Bernoulli Equation with no velocity potential & conservative force |  |  |

# Energy Equations

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| Heading | Definition’s | Formulae’s |
| Basic Statement | The rate of change of total energy (kinetic, potential etc) on a given segment of a compressible inviscid fluid as it moves is equal to the total work done by the pressure on the boundary provided | Let T=Kinetic Energy, P=Potential, I=internal, W=work done, P=pressure =Potenital field  where |

# 2-D Flow

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| Heading | Definition’s | Formulae’s |
| Stream Function | Stream function signifies that the difference of its values across two points represent the flow across any line joining the points (inform of flux) | is velocity vector.  Let be the potential |
| Flux |  | *Flux flowing is equal to*  *Flux across a small segment* |
| Velocity in Polar Coordinates |  |  |
| Complex Potential | Complex potential is potential function + I \* stream function |  |
| Spin Component of q |  |  |
| Velocity in complex plane |  |  |
| Strength Source and Sink | A source of strength m in 2D is such that the flow across any small curve surrounding it is |  |
| Dipole | A combination of source and sink. is the strength of the doublet represented as , is the distance | *Doublets At point ,* |
| Image of a source | In a fluid, if there exists a surface S where there is no flow, then the system of sources, sinks and doublets on opposite sides of S is known as the system with regards to that curve | 1. *F*or St line *Direct image* 2. *For circle Source at inverse point and sink at centre* |
| Milne Thompson Theorem | Let f(z) be the complex potential in a flow having no rigid boundaries such that there are no singularities inside circle |z|=a. Then if we introduce a solid cylinder at |z|=a, then the new complex potential would be |  |
| Blasius Theorem | In steady 2D Irrotational flow of an incompressible liquid with complex potential w=f(z), the pressure thrust of a cylinder of any shape is represented by Force(X,Y) and the moment M | *Where C is contour.* |
| Flow | Value of integral is called flow from a to b. |  |
| Kelvin Circulation Theorem | When the external forces are conservative and derived from single valued potential function and density is a function of pressure only. Then the circulation in a particular closed circuit is constant all the time. |  |
| Green’s Theorem | Let two single valued continuous function and |  |
| Kelvin Minimum Energy Theorem | The Irrotational motion of a liquid occupying a simply connected region would have less kinetic energy than any other motion having same normal velocity at the boundary. |  |

# Motion of Cylinders

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| Heading | Definition’s | Formulae’s |
| Stream function for pure rotational motion |  | *where is the angular velocity* |
| Kinetic Energy |  |  |
| When cylinder is moving in an infinite mass of liquid in x dirn |  | *Solution of the form* |
| Fluid flowing at vel=-U |  |  |
| Cylinder moving with velocity U |  |  |
| Complex potential due to circulation |  |  |
| Streaming & Circulation around a fixed cylinder |  |  |

# Aerofoils & Naiver Stokes Equation

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| Heading | Definition’s | Formulae’s |
| Conformal Mapping | Preserves angle and side ratios | *Area Ratio ( to z) equals to*  *Velocity Ratio equals to 1/|f’(z)|*  *Kinetic Energy Preserved*  *Complex potential preserved* |
| Differentiation |  |  |
| Joukowski’s Transformation |  |  |
| Kutta- Joukowski Theorem | When a cylinder of any shape is placed in uniform stream of speed U then the resultant thrust on the cylinder is per unit length normal to the cylinder, where K is the circulation. | *with the positive x axis, other variables have their usual meanings* |
| Flow past a circle |  | *d*irn with the -ive x axis |
| Flow past an aerofoil |  |  |
| Naiver Stokes Equation |  |  |
| Constitutive law of Newtonian compressible flow |  | *Where i, j= x,,y,z* |
| Naiver Stokes for incompressible fluid |  |  |
| Dissipation of energy |  |  |

# Solving Differential Equations

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| Heading | Definition’s | Formulae’s |
| Generic Way to find PI |  |  |

Stream function page 126